

## Computation of corona inception voltage by the charge simulation method

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**Abstract:** In this paper, the electric field distribution and corona inception voltage of the hemispherically tipped rod-plane electrode system have been computed by a newly developed software based on the charge simulation method. An accurate physical model for the primary electron avalanche has been used. Space charges have been simulated by ring charges. Visual Basic 6.0 has been used to develop the software. The software runs under Windows operating system and is user friendly.

### 1. Introduction

High voltage equipment is the backbone of modern power systems. Besides generation, transmission and distribution of electrical energy, high voltages are also extensively used for many industrial, scientific and engineering applications such as electrostatic precipitators for the removal of dust from flue gases, atomization of liquids, paint spraying and pesticide spraying, ozone generation of water and sewage treatment, X-Ray generators and particle accelerators, high-power lasers and ion beams, plasma sources for semiconductor manufacture, superconducting magnet coils.

In all such applications, the insulation of the high voltage conductor is of primary importance. For the proper design, safe and reliable operation of the insulation system, a complete knowledge of the electric field distribution of any high voltage device is required.

Insulating materials play a critical role in the design and performance of high voltage power equipments. Gases are the simplest and the most widely used dielectrics. Air is used as an insulator for outdoor as well as indoor high voltage power networks. For insulation purposes, it is used to provide the phase to phase as well as phase to ground insulation. In addition, air is also used in chopping, spark and measurement gaps.

Early investigations of electrical breakdown of gases have been made nearly at the end of nineteenth century [1]. In the investigations of electrical breakdown of gases, particularly in the study of corona phenomena in atmospheric air, the asymmetrical nonuniform field configuration has been a valuable tool for experimental observations due to the local confinement of pre-breakdown ionization around the stressed electrode [2]. Many of the basic breakdown studies were carried out with rod-to-plane electrodes or with two rods. For the sake of result comparison and repeatability, the rod electrode chosen

was standardized as a cylindrical shaft with a hemispherical tip of equal diameter.

In this paper, the electric field and potential distribution in the gap between a cylindrical rod having a hemispherical tip and an infinite plane perpendicular to the cylinder axis have been computed and corona inception voltage of this electrode system has been computed using newly developed software based on the charge simulation method.

Visual Basic 6.0 was used to develop the software. The software runs under Windows operating system and has a user-friendly interface.

### 2. Electric field calculation by charge simulation method (CSM)

Electric fields of nonuniform electrode systems can be calculated by numerical methods such as Finite Element, Boundary Element or Charge Simulation Method (CSM). In this paper Charge Simulation Method has been used to calculate the electric field distribution [3-8].

In the Charge Simulation Method, the actual electric field is simulated by a number of discrete simulation charges that are located inside the conductors. Values of simulation charges are determined by satisfying the boundary conditions at a number of contour points selected at the conductor surfaces. Once the values of simulation charges are determined, then the potential and electric field of any point in the region outside the conductors can be calculated using the superposition principle as follows: if several discrete charges of any type (point, line or ring) are present in a conductor, the potential at any point at the surface of the conductor can be calculated by the summation of the potential contribution of all the individual simulation charges using the equation (1)

$$V_i = \sum_{j=1}^n p_{ij} \cdot q_j \quad i = j = 1, \dots, n \quad (1)$$

where  $p_{ij}$  is the potential coefficient related to the potential of the  $j^{\text{th}}$  charge at the  $i^{\text{th}}$  point  $q_j$  is the simulation charge,  $n$  is the number of charge. If the number of contour points is selected as equal to the number of simulation charges, a set of linear equations for the potentials of contour points can be given by

$$[p] \cdot [q] = [V] \quad (2)$$

where [p] is the potential coefficients matrix, [q] is a column vector for simulation charges, and [V] is a column vector for potentials of contour points. Since potentials of contour points are known and equal to the applied voltage of the conductors, the values of simulation charges are calculated first by solving the equation (2). Electric field is calculated by vectorial superposition of magnitudes of its various directional components.

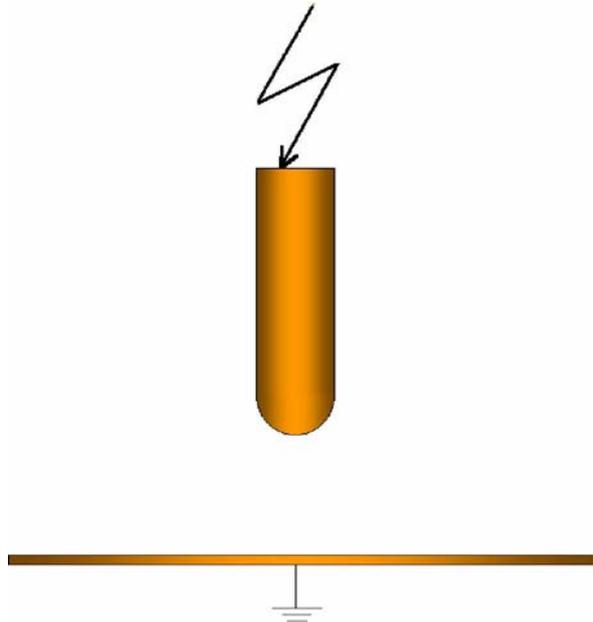


Figure 1 Rod-plane electrode system.

### 3. Computation of corona inception voltage

In the calculations of corona inception voltage a few assumptions have been made. The avalanche grows from a single electron along the vertical axis and this electron starts from that point at which electric field value is 24.4 kV/cm at normal pressure (1.013 bar). An avalanche starting from anywhere else in the gap has little chance to grow enough to initiate a streamer. The avalanche head moves to the anode under the influence of the electrostatic field only [5].

There are many expressions relating to Townsend first ionization coefficient  $\alpha$  to the field strength  $E$  and relative air density  $\delta$ . The following expression has been used for  $\alpha$

$$\frac{\alpha}{\delta} = K.A \left( \frac{E}{\delta} - B \right)^2 \quad (3)$$

where  $K=18$  (Schumann's condition for corona onset)  $A=0.02345$  cm/kV and  $B=24.4$  kV/cm.

Setting  $\delta = 1$  (at normal conditions) and  $K = 18$  gives

$$\alpha = 0.4221 \cdot (E - 24.4)^2 \quad (4)$$

In this paper  $K$  has been calculated with the Simpson method.

$$K = \int_{z_{cr}}^a \alpha dz \quad (5)$$

where  $z_{cr}$  is the critical field point and  $a$  is the electrode gap. The number of electrons at the avalanche head is

$$n_e = \text{Exp} \left( \int_{z_{cr}}^a \alpha dz \right) \quad (6)$$

and the total number of positive ions at the avalanche is

$$n_+ = \left( \text{Exp} \left( \int_{z_{cr}}^a \alpha dz \right) \right) - 1 = n_e - 1 \quad (7)$$

The ring charges for the positive ions have been located at the points where the collisions take place and since the velocities of electrons are 1000 times higher than the velocities of positive ions, the ring charges assumed fixed at their positions. The radius of ring charge is the function of the electron diffusion coefficient  $D_e$  and of electron drift velocity. Assuming cylindrical diffusion it is expressed as

$$r = (4 \cdot D_e \cdot t)^{1/2} \quad (8)$$

where  $D_e=430$  cm<sup>2</sup>/s for air at normal conditions,  $t$  is the transit time of an electron between the critical field point and the collision point. It has been obtained by dividing the distance crossed by the drift velocity  $v_e$  expressed as

$$v_e = (2.74 \frac{E}{p} + 39.1) 10^5 \text{ (cm/s)} \quad (9)$$

where  $E/p$  in V/cm and  $p$  is the air pressure.

After every collision the charge content of the circular ring is equal to the charge quantity of positive ions formed by this process.

If the conditions for corona inception are not satisfied, then the applied voltage is altered and the procedure is repeated

### 4. Corona 2002 Software

A user-friendly software named Corona 2002 has been developed using Microsoft Visual Basic 6.0. Corona 2002 runs under Windows operating system on an ordinary PC and is easy to use it.

Charge Simulation Method has been used to determine the electric field distribution and corona inception voltage. Calculated plotted equipotential lines have been colored according to their values (Figure 3). Potential errors at the control points have been used to control the accuracy of the simulation.

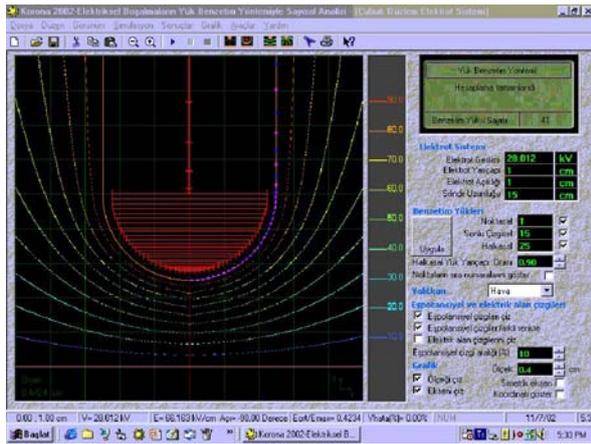


Figure 3. Corona 2002 Software user interface and electric field distribution of rod-plane electrode system.

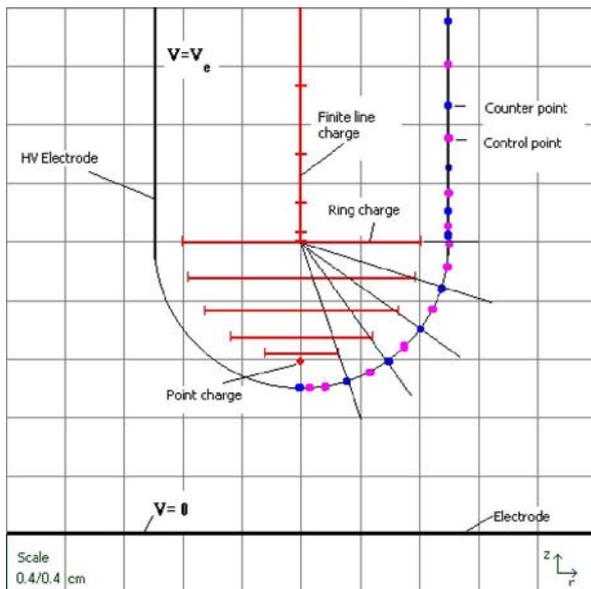


Figure 4. Positions of ring and finite line charges, and their corresponding contour and control points at boundary of the rod electrode.

Simulation of rod electrode has been made by a point charge located near the hemispherical tip on the vertical axis, a number of ring charges located in the hemispherical cap and a number of finite line charges located on the vertical axis in the cylindrical shaft (Figure 4). Finite line charge lengths were not equal. Their lengths were shorter near the tip but longer towards the tail. The number of counter points and the number of control points have been chosen equal to the number of charges. The number and position of charges have significant influence on the accuracy of the simulation. The radiuses of ring charges also influence the accuracy significantly. The positions of the contour points play an important role on the accuracy. Accurate solution can be obtained when the contour points have been chosen on the same

orthogonal line starting from the centre of the hemispherical tip and passing at the end point of each ring charge (Fig. 4). Control points have been chosen at the middle of two adjacent contour points on the electrode boundary. Grounded infinite plane electrode has been simulated by taking the images of the simulation charges inside the rod electrode. Gaussian Elimination Method has been used to solve the systems of linear equations. Approximate formulations have been used for calculating the complete elliptic integral of the first kind and complete elliptic integral of the second kind in the potential and electric field equations of the ring charge [7].

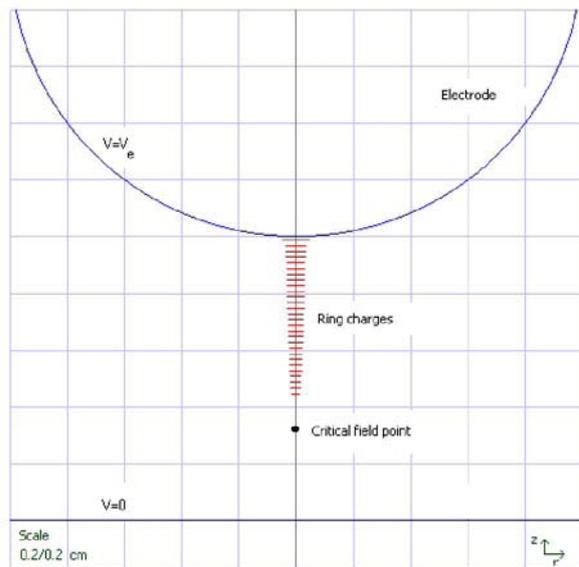


Figure 5. Positions of ring charges for the simulation of positive ions at the primary avalanche, and critical field point at which a single electron starts to navigate towards the rod electrode.

Table 1 shows the computation results for the rod electrode radius of 0.1cm and applied voltage of 10 kV, according to various electrode gaps.

Table 1. Maximum electric field and utility factor values of various electrode gap geometries ( $R=0.1$  cm,  $V_e=10$  kV)

a(cm)	E(kV/cm)	Utility factor ( $\eta$ )
0.5	101.45	0.1971
1.0	90.52	0.1105
1.5	86.59	0.0770
2.0	84.47	0.0592
2.5	83.25	0.0480

Figure 5 shows the variation of maximum electric field versus electrode gap for electrode radius of 0,1 cm and applied voltage of 10kV. The variation of maximum electric field versus electrode radius have also been calculated and plotted. Critical field point

has been calculated as  $z_{cr}=0.9040998$  cm where electrode gap is 1 cm, electrode radius is 0.1cm and applied voltage is 10 kV. Calculated corona inception voltage is 37.005795 kV and critical field point is 0.6399 cm for the gap length of 1 cm and electrode radius of 0.1 cm and is 25.37 kV at 1 cm radius.

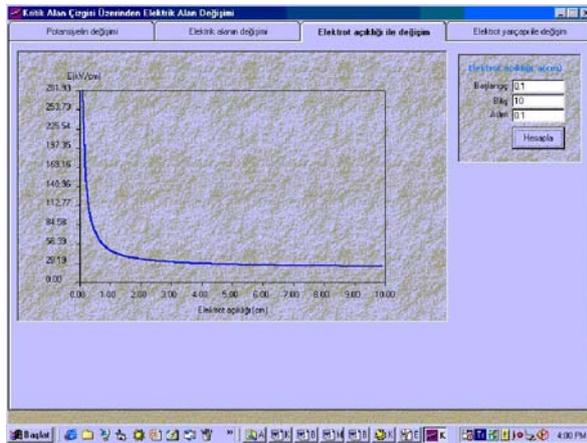


Figure 5. Variation of maximum electric field versus electrode gap ( $R=0,1$  cm,  $V_e=10$  kV).

The variation of utility factor versus electrode gap has been shown in Figure 6.

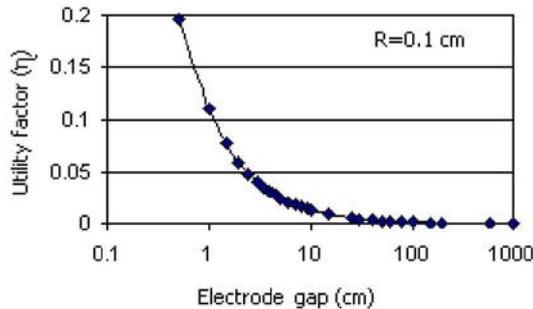


Figure 6. Variation of utility factor versus electrode gap ( $R=0,1$  cm,  $V_e=10$  kV).

## 5. Conclusion

A new software has been developed and electric field distribution and corona inception voltage has been calculated for rod plane electrode system. The software runs under Windows operating system and is user-friendly. Results are in good agreement with those found in the literature.

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